


SOALAN RAMALAN

MATEMATIK TAMBAHAN

KERTAS 1

PENGAMIRAN
INTEGRATION

SIR VEN
(GURU ADIWIRA KEBANGSAAN 2019)

SPM 2023

PENGAMIRAN
INTEGRATION

It is given that
Diberi bahawa

$$\int \frac{4}{(3x+1)^a}\,dx = \frac{b}{(3x+1)^7} + c,$$

where a, b and c are constants. Find the value of a and of b.
dengan keadaan a, b dan c ialah pemalar. Cari nilai a dan nilai b.

[3 *marks* / *markah*]

Diagram 2 shows part of the curve $y = g(x)$.
Rajah 2 menunjukkan sebahagian daripada lengkung $y = g(x)$.

The shaded region is defined as $\int_d^c g(x)\,dx = 9$.

Kawasan berlorek ditakrifkan sebagai $\int_d^c g(x)\,dx = 9$.

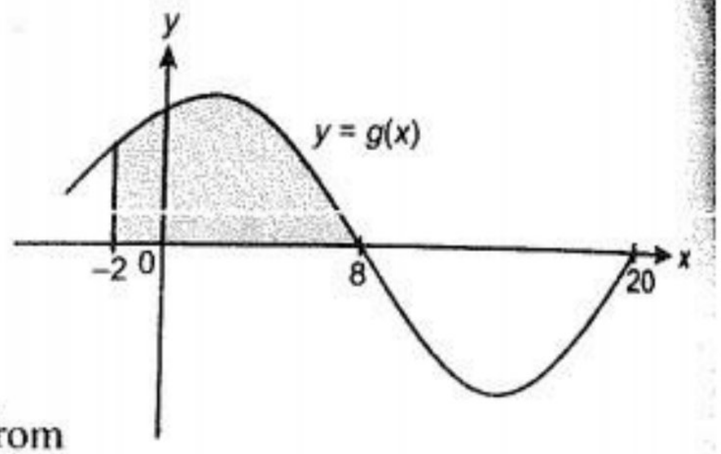

Diagram 2 / *Rajah 2*

(a) State the value of c and of d.
Nyatakan nilai c dan nilai d.

(b) It is given that the area bounded by $y = g(x)$ and the x-axis from $x = -2$ to $x = 20$ is 19. State the value of $\int_8^{20} g(x)dx$.
Diberi bahawa luas kawasan yang dibatasi oleh $y = g(x)$ dan paksi-x dari $x = -2$ ke $x = 20$ ialah 19. Nyatakan nilai $\int_8^{20} g(x)dx$.

[2 *marks* / *markah*]

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Given / *Diberi* $\int \frac{8}{(1-3x)^2}\,dx = j(1-3x)^k + c$

Find the value of j and k.
Cari nilai j dan k.

[3 *marks / markah*

Answer / *Jawapan* :

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Given / *Diberi*

$\int_0^2 h(x)dx = a$ and / *dan* $\int_0^2 [4x + bh(x)]dx = 16$,

express b in terms of a.
ungkapkan b dalam sebutan a.

[3 *marks* / *markah*]

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)Given that $\int_{-1}^{4} g(x)\,dx = 3$. Find the value of k if $\int_{-1}^{4} [4x + kg(x)]\,dx = 10$.

Diberi bahawa $\int_{-1}^{4} g(x)\,dx = 3$. *Cari nilai k jika* $\int_{-1}^{4} [4x + kg(x)]\,dx = 10$.

[3 *marks / markah*]

Answer / *Jawapan* :

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Diagram 8 shows a shaded region bounded by the graph $y = h(x)$ and x-axis from $x = -a$ to $x = -b$.
Rajah 8 menunjukkan kawasan berlorek yang dibatasi oleh graf $y = h(x)$ dan paksi-x dari $x = -a$ ke $x = -b$.

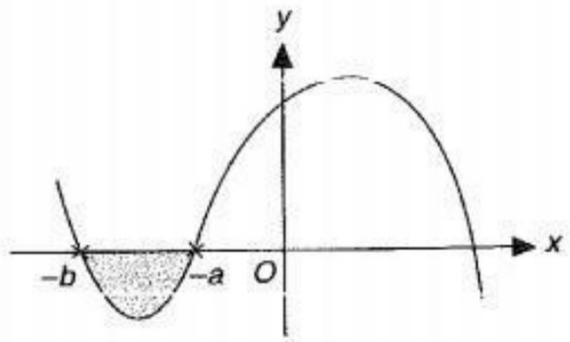

Diagram 8
Rajah 8

(a) It is given that $\frac{d}{dx}[2g(x)] = h(x)$ and $[g(x)]_{-a}^{-b} = -9$, state the area of the shaded region.

Diberi bahawa $\frac{d}{dx}[2g(x)] = h(x)$ dan $[g(x)]_{-a}^{-b} = -9$, nyatakan luas kawasan berlorek.

(b) The graph $y = h(x)$ passes through point (4, 21). Given $\frac{d}{dx}[h(x)] = 3x^2 - 4x + 5$, *find $h(x)$ in terms of x.*

Graf $y = h(x)$ melalui titik (4, 21). Diberi $\frac{d}{dx}[h(x)] = 3x^2 - 4x + 5$, cari $h(x)$ dalam sebutan x.

[4 marks]
[4 markah]

[4 marks]
[4 markah]

)Given that $\int_0^1 \frac{1}{(2x-1)^2}\,dx = 5k - 2$, where k is a constant, find the value of k.

Diberi $\int_0^1 \frac{1}{(2x-1)^2}\,dx = 5k - 2$, *di mana k ialah pemalar, cari nilai bagi k.*

[3 *marks*]
[3 *markah*]

Answer/*Jawapan*:

Given $\int_{-1}^{k} (2x - 10)\, dx = -36$, find the value of k.

Diberi, $\int_{-1}^{k} (2x - 10)\, dx = -36$, *cari nilai* k.

[3 *marks*/3 *markah*]

Given $\int \frac{8}{(2x-9)^n}\,dx = \frac{r}{(2x-9)^4} + c$, where c, n and r are constants. Find the values of n and r.

Diberi $\int \frac{8}{(2x-9)^n}\,dx = \frac{r}{(2x-9)^4} + c$, *dengan keadaan* c, n *dan* r *ialah pemalar. Cari nilai n dan nilai r.*

[3 *marks*/3 *markah*]

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Given $y = \frac{k}{(4-3x)^3}$ and $\frac{dy}{dx} = f(x)$, find the value of k if $\int_1^2 [f(x) + 1]\,dx = 10$.

Diberi $y = \frac{k}{(4-3x)^3}$ dan $\frac{dy}{dx} = f(x)$, cari nilai bagi k jika $\int_1^2 [f(x) + 1]\,dx = 10$.

Answer / Jawapan :

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Given that $\frac{d}{dx}\left[\frac{x+3}{(1+2x)^2}\right] = 2 - f(x)$, find, $\int_1^2 f(x)\,dx$

Diberi $\frac{d}{dx}\left[\frac{x+3}{(1+2x)^2}\right] = 2 - f(x)$, *cari*, $\int_1^2 f(x)\,dx$.

PENGAMIRAN
INTEGRATION

Given $\frac{d}{dx}\left[\frac{2x^2}{3x-2}\right] = 3f(x)$, find the value of $\int_1^3 \frac{1}{2} f(x)\, dx$.

Diberi $\frac{d}{dx}\left[\frac{2x^2}{3x-2}\right] = 3f(x)$, *cari nilai bagi* $\int_1^3 \frac{1}{2} f(x)\, dx$

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Given that $y=\frac{2x}{(x+3)^3}$ and $\frac{dy}{dx}=\frac{6-4x}{(x+3)^4}$, find $\int_{-1}^{0}\frac{3-2x}{(x+3)^4}dx$.

Diberi bahawa $y=\frac{2x}{(x+3)^3}$ *dan* $\frac{dy}{dx}=\frac{6-4x}{(x+3)^4}$, *cari* $\int_{-1}^{0}\frac{3-2x}{(x+3)^4}dx$.

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Diberi bahawa $y = \dfrac{p}{(4-3x)^3}$ dan $\dfrac{dy}{dx} = f(x)$, cari nilai p jika

$$\int_1^2 [f(x)+1]\,dx = 10.$$

Given that $y = \dfrac{p}{(4-3x)^3}$ *and* $\dfrac{dy}{dx} = f(x)$,

find the value of p if $\int_1^2 [f(x)+1]\,dx = 10.$

10 (a) Diberi bahawa $\int_4^6 n(x)dx = 13$, cari

Given that $\int_4^6 n(x)dx = 13$, *find*

(i) $\int_6^4 3n(x)dx$

(ii) $\int_4^5 n(x)dx + \int_5^6 [n(x)-2]dx$

[3 *markah*]
[3 *marks*]

(b) Cari $\int_5^m (8x-5)dx$ dalam sebutan m.

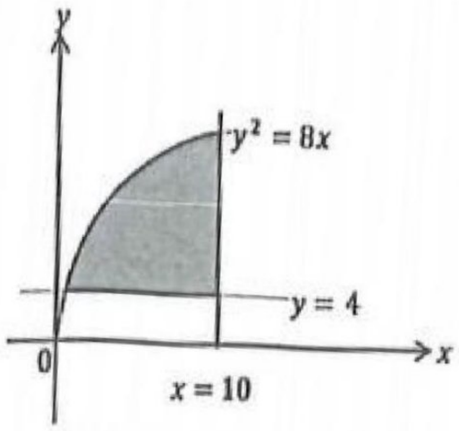
Find $\int_5^m (8x-5)dx$ *in terms of* m.

[2 *markah*]
[2 *marks*]

Jawapan / *Answer*:

14 (a) Diberi fungsi kecerunan bagi suatu lengkung ialah $4x-10$ dan lengkung itu melalui titik (8,4). Cari persamaan lengkung itu.
[4 *markah*]
Given that the gradient function of a curve is $4x-10$ *and the curve passes through point* (8 ,4). *Find the equation of the curve.*
[4 *marks*]

(b) Rajah 14(b) menunjukkan lengkung $y^2=8x$, garis lurus $y=4$ dan $x=10$.
Diagram 14(b) *shows the curve* $y^2=8x$, *the straight lines* $y=4$ *and* $x=10$.



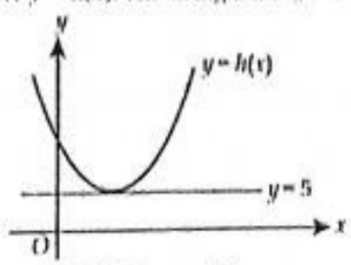
Rajah 14(b) / *Diagram* 14(b)

Cari luas rantau berlorek.
[4 *markah*]
Find the area of the shaded region.
[4 *marks*]

Jawapan / *Answer*:

PENGAMIRAN
INTEGRATION

14 (a) Rajah 14(a) menunjukkan lengkung $y = h(x)$. Garis lurus $y = 5$ ialah tangen kepada lengkung itu.
Diagram 14 (a) *shows the curve* $y = h(x)$. *The straight line* $y = 5$ *is the tangent to the curve.*

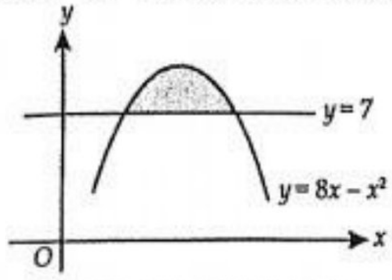


Rajah 14/ *Diagram* 14

Diberi fungsi kecerunan bagi lengkung itu ialah $2x-4$, cari persamaan bagi lengkung itu.
Given that the gradient function of the curve is $2x-4$, *find the equation of the curve.*

[4 *markah*]
[4 *marks*]

(b) Rajah 14(b) menunjukkan lengkung $y = 8x - x^2$ dan garis lurus $y = 7$.
Diagram 14(b) *shows the curve* $y = 8x - x^2$ *and the straight line* $y = 7$.



Rajah 14(b)/ *Diagram* 14(b)

Cari luas, dalam unit^2, bagi rantau berlorek.
Find the area of the shaded region, in unit^2.

[4 *markah*]
[4 *marks*]

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(a) Cari nilai bagi [3 *markah*]
Find the value of [3 *marks*]

$$\int_{-\frac{7}{2}}^{-3} (2p+7)^3 dp$$

(b) Diberi suatu lengkung $y = g(x)$. Persamaan tangen kepada lengkung itu ialah $y = 10$.
Diberi $g'(x) = -x + 5$, cari persamaan lengkung itu. [4 *markah*]
Given a curve $y = g(x)$. *The equation of the tangent to the curve is* $y = 10$.
Given that $g'(x) = -x + 5$, *find the equation of the curve.* [4 *marks*]

Jawapan/ *Answer*:

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INTEGRATION

$$\int \frac{4}{(3x+1)^a}\,dx = 4\int (3x+1)^{-a}\,dx$$
$$= 4\left[\frac{(3x+1)^{-a+1}}{3(-a+1)}\right] + c$$
$$4\left[\frac{(3x+1)^{-(a-1)}}{3(-a+1)}\right] + c = \frac{b}{(3x+1)^7} + c$$
$$\frac{4}{3(1+a)}\left(\frac{1}{(3x+1)^{a-1}}\right) + c = \frac{b}{(3x+1)^7} + c$$
$$a - 1 = 7$$
$$a = 8$$
$$b = \frac{4}{3(1-a)}$$
$$= \frac{4}{3(1-8)}$$
$$= -\frac{4}{21}$$

(a) From graph, $c = 8$, $d = -2$

(b) $\int_8^{20} g(x)\,dx = \int_{-2}^{20} g(x)\,dx - \int_{-2}^{8} g(x)\,dx$
$= 19 - 9$
$= 10$

$$\int \frac{8}{(1-3x)^2}\,dx = j(1-3x)^k + c$$

Let $u = 1 - 3x$

$$\frac{du}{dx} = -3$$
$$dx = -\frac{du}{3}$$
$$\int \frac{8}{(1-3x)^2}\,dx = \int \frac{8}{u^2}\left(-\frac{du}{3}\right)$$
$$= -\frac{8}{3}\left[\frac{u^{-2+1}}{-2+1}\right] + c$$
$$= -\frac{8}{3}\left[\frac{u^{-1}}{-1}\right] + c$$
$$= \frac{8}{3}u^{-1} + c$$
$$= \frac{8}{3}(1-3x)^{-1} + c$$

$$\frac{8}{3}(1-3x)^{-1} + c = j(1-3x)^k + c$$
$$\therefore j = \frac{8}{3},$$
$$k = -1$$

$$\int_0^2 [4x + bh(x)]dx = 16$$
$$\int_0^2 4x\,dx + b\int_0^2 h(x)dx = 16$$
$$\left[\frac{4x^2}{2}\right]_0^2 + ba = 16$$
$$2(2)^2 - 0 + ba = 16$$
$$8 + ba = 16$$
$$ba = 16 - 8$$
$$b = \frac{8}{a}$$

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$$\int_{-1}^{4}[4x + kg(x)]dx = 10$$

$$\int_{-1}^{4} 4x\,dx + \int_{-1}^{4} kg(x)\,dx = 10$$

$$\left[\frac{4x^2}{2}\right]_{-1}^{4} + k\int_{-1}^{4} g(x)\,dx = 10$$

$$[2(4)^2 - 2(-1)^2] + 3k = 10$$

$$30 + 3k = 10$$

$$3k = -20$$

$$\therefore k = -\frac{20}{3}$$

(a) Area of shaded region $= \int_{-6}^{-5} h(x)\,dx$

$= [2g(x)]_{-6}^{-5}$

$= 2(-9)$

$= -18$

Area = 18 units^2

(b) $h(x) = \int 3x^2 - 4x + 5\,dx$

$= \frac{3x^3}{3} - \frac{4x^2}{2} + 5x + c$

$h(x) = x^3 - 2x^2 + 5x + c$

Since $h(x)$ passes through (4, 21), $x = 4$ and $h(x) = 21$.

$21 = 64 - 2(16) + 20 + c$

$c = 21 - 64 + 32 - 20$

$= -31$

$\therefore h(x) = x^3 - 2x^2 + 5x - 31$

$$\int_0^1 \frac{1}{(2x-1)^2}\,dx = \int_0^1 (2x-1)^{-2}\,dx$$

$$= \left[\frac{(2x-1)^{-1}}{2(-1)}\right]_0^1$$

$$= \left[\frac{1}{-2(2x-1)}\right]_0^1$$

$$= \frac{1}{-2(2(1)-1)} - \frac{1}{-2(2(0)-1)}$$

$$= \frac{1}{-2} - \frac{1}{2}$$

$$= -1$$

$5k - 2 = -1$

$5k = 1$

$k = \frac{1}{5}$

$$\int_{-1}^{k} (2x - 10)\,dx = -36$$

$$\left[x^2 - 10x\right]_{-1}^{k} = -36$$

$$(k^2 - 10k) - [(-1)^2 - 10(-1)] = -36$$

$$k^2 - 10k - 11 = -36$$

$$k^2 - 10k + 25 = 0$$

$$(k - 5)(k - 5) = 0$$

$$k = 5$$

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$$\int \frac{8}{(2x-9)^n}\,dx = \int 8(2x-9)^{-n}\,dx$$

$$= \frac{8(2x-9)^{-n+1}}{2(-n+1)}$$

$$= \frac{4}{(-n+1)(2x-9)^{n-1}}$$

$$n-1 = 4$$

$$n = 5$$

$$r = \frac{4}{-5+1}$$

$$= -1$$

$$\int_1^2 f(x)\,dx + \int_1^2 1\,dx = 10$$

$$\left[\frac{k}{(4-3x)^3}\right]_1^2 + [x]_1^2 = 10$$

$$\left[\frac{k}{[4-3(2)]^3} - \frac{k}{[4-3(1)]^3}\right] + [2-1] = 10$$

$$\left[\frac{k}{-8} - \frac{k}{1}\right] + 1 = 10$$

$$\frac{9}{-8}k = 9$$

$$k = -8$$

$$\frac{d}{dx}\left[\frac{x+3}{(1+2x)^2}\right] = 2 - f(x)$$

$$\int_1^2 2 - f(x)\,dx = \left[\frac{x+3}{(1+2x)^2}\right]_1^2$$

$$[2x]_1^2 - \int_1^2 f(x)\,dx = \left[\frac{x+3}{(1+2x)^2}\right]_1^2$$

$$\int_1^2 f(x)\,dx = [2x]_1^2 - \left[\frac{x+3}{(1+2x)^2}\right]_1^2$$

$$= [4-2] - \left[\frac{1}{5} - \frac{4}{9}\right]$$

$$= 2 - \left[-\frac{11}{45}\right]$$

$$= 2\frac{11}{45}$$

$$\frac{d}{dx}\left[\frac{2x^2}{3x-2}\right] = 3f(x)$$

$$\frac{1}{6}\int_1^3 3f(x)\,dx = \frac{1}{6}\left[\frac{2x^2}{3x-2}\right]_1^3$$

$$\int_1^3 \frac{1}{2}f(x)\,dx = \frac{1}{6}\left[\left(\frac{2(3)^2}{3(3)-2}\right) - \left(\frac{2(1)^2}{3(1)-2}\right)\right]$$

$$= \frac{1}{6}\left[\frac{18}{7} - 2\right]$$

$$= \frac{1}{6}\left(\frac{4}{7}\right)$$

$$= \frac{2}{21}$$

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m1

$\frac{dy}{dx} = \frac{(-4x}{(x+3)^3}$

$y = \int \frac{(-4x}{(x+3)^3}\, dx$

$\frac{2x}{(x+3)^3} = \int \frac{2(3-2x)}{(x+3)^3}\, dx$

$\frac{x}{(x+3)^3} = \int \frac{3-2x}{(x+3)^3}\, dx$

$\int_{-1}^{0} \frac{3-2x}{(x+3)^3}\, dx = \left[\frac{x}{(x+3)^3}\right]_{-1}^{0}$

$= \frac{0}{(0+3)^3} - \left(\frac{-1}{(-1+3)^3}\right)$

$= 0 - \left(\frac{-1}{8}\right)$

$= \frac{1}{8}$

$y = \int f(x)\, dx = \frac{p}{(4-3x)^3}$

$\therefore \int_1^2 f(x)\, dx + \int_1^2 1\, dx = 10$

$\left[\frac{p}{(4-3x)^3}\right]_1^2 + [x]_1^2 = 10$

$\left(\frac{p}{(-2)^3} - \frac{p}{1^3}\right) + 1 = 10$

$\left(\frac{p}{-8} - \frac{p}{1}\right) = 9$

$\frac{-9}{8}p = 9$

$p = -8$

10 (a)(i)	$3(-\int_4^6 n(x)dx) = 3(-13)$ $= -39$	1
10 (a)(ii)	$\int_4^5 n(x)dx + \int_5^6 n(x)dx - \int_5^6 2\, dx$	
	$\int_4^6 n(x)dx - [2x]_5^6$	1
	$13 - [2(6) - 2(5)]$	
	11	1
10 (b)	$[4x^2 - 5x]_5^m$	1
	$(4m^2 - 5m) - [4(5^2) - 5(5)]$ $4m^2 - 5m - 75$	1

14 (a)	$\frac{dy}{dx} = 4x - 10$	
	$y - \int 4x - 10dx$	
	$y = 2x^2 - 10x + c$	1
	$4 = 2(8)^2 - 10(8) + c$	1
	$c = -44$	
	$y = 2x^2 - 10x - 44$	1
14 (b)	Titik persilangan $4^2 = 8x$	
	$x = 2$	1
	Luas kawasan berlorek	
	$\int_2^{10} (8x)^{\frac{1}{2}}\, dx - 8\times4$	1
	$= \left[\frac{2(8x)^{\frac{3}{2}}}{3(8)}\right]_2^{10} - 32$	1
	$= \left(\frac{2(8(10))^{\frac{3}{2}}}{3(10)}\right) - \left(\frac{2(8(2))^{\frac{3}{2}}}{3(2)}\right) - 32$	1
	-22.30	1

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14 (a)	$\frac{dy}{dx} = 2x - 4 = 0$ $x = 2$ Titik minimum = (2, 5) $y = \int 2x - 4\,dx$ $y = x^2 - 4x + c$ $5 = 2^2 - 4(2) + c$ $y = x^2 - 4x + 9$	1 1 1 1	
14 (b)	$8x - x^2 = 7$ $x^2 - 8x + 7 = 0$ $x = 1 \quad x = 7$ $\left[4x^2 - \frac{x^3}{3}\right]_1^7 - (6 \times 7)$ $\left[4(7)^2 - \frac{7^3}{3}\right] - \left[4(1)^2 - \frac{1^3}{3}\right] - 42$ 36	1 1 1 1	